Discussion Problems 4

Problem One: Infinity Plus One

Let ★ be any value that isn't a natural number. Prove that $|\mathbb{N} \cup \{\bigstar\}| = |\mathbb{N}|$.

Problem Two: Propositional Equivalences

In lecture, we saw two different definitions of antisymmetry. If R is a binary relation over a set A, we said that R is antisymmetric iff

For any $x, y \in R$: if xRy and yRx, then $x = y$

We also said R is antisymmetric iff

For any $x, y \in R$: if xRy and $x \neq y$, then the relation yRx does not hold.

These statements are equivalent because the propositional formula $p \land q \to r$ is equivalent to the propositional formula $p \land \neg r \to \neg q$. Prove this by using a truth table.

Problem Three: Translating into Logic

i. Given the predicate *Person*(x), which states that x is a person, and *Muggle*(x), which states that x is a muggle, write a statement in first-order logic that says "some (but not all) people are muggles."

ii. Given the predicate *Person*(x), which states that x is a person, and *Commoner*(x), which states that x is a commoner, write a statement in first-order logic that says "there are either zero or one people who are not commoners."